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Trophic analysis of a historical network reveals temporal information

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Abstract

Trophic analysis exposes the underlying hierarchies present in large complex systems. This allows one to use data to diagnose the sources, propagation paths, and basins of influence of shocks or information among variables or agents, which may be utilised to analyse dynamics in social, economic and historical data sets. Often, the analysis of static networks provides an aggregated picture of a dynamical process and explicit temporal information is typically missing or incomplete. Yet, for many networks, particularly historical ones, temporal information is often implicit, for example in the direction of edges in a network. In this paper, we show that the application of trophic analysis allows one to use the network structure to infer temporal information. We demonstrate this on a sociohistorical network derived from the study of hadith, which are narratives about the Prophet Muhammad's actions and sayings that cite the people that transmitted the narratives from one generation to the next before they were systematically written down. We corroborate the results of the trophic analysis with a partially specified time labelling of a subset of the transmitters. The results correlate in a manner consistent with an observed history of information transmission flowing through the network. Thus, we show that one may reconstruct a temporal structure for a complex network in which information diffuses from one agent to another via social links and thus allows for the reconstruction of an event based temporal network from an aggregated static snapshot. Our paper demonstrates the utility of trophic analysis in revealing novel information from hierarchical structure, thus showing its potential for probing complex systems, particularly those with an inherent asymmetry.

Keywords: Social network, Historical network, Temporal network, Trophic analysis

Introduction

Networks are an especially powerful way to represent complex systems. They constitute many of the systems that underlie much of our infrastructure and social interactions, as well as ecological and biological systems that control and regulate life. While the underlying architecture of many networks is usually static, it is often connected to numerous dynamical processes that are the results of the functional processes that occur on it (Barrat et al. 2008). Moreover, many complex systems exist diachronically and thus their structures are in flux. For these reasons, a robust network analysis of such complex systems requires temporal information in order to account for this dynamic element. This paper demonstrates the viability of applying trophic analysis, a method originally

devised to quantify the hierarchy of species in a food chain (Levine 1980), to extract temporal information of nodes in a directed network. We apply trophic analysis to a historical social network that we created from a historical textual corpus. This network was devoted to the preservation and dissemination of narratives about, and statements from, the Prophet Muhammad, known as hadiths. This network emerged in the 7th century and lasted nine centuries thereafter.

We show how trophic analysis provides accurate temporal information on the nodes in the network by corroborating it with information about the nodes obtained from outside of the network, thus proving their effectiveness in extracting temporal information from the graph's structure. The fact that trophic analysis can effectively extract temporal information from hierarchical structures proves its utility for network scientists interested in probing hierarchical structure further and utilising asymmetrical structure to reveal novel insights into complex systems. For scholars of hadith and historians of early Islam, which often wrestle with incomplete temporal information on individuals involved in the preservation and dissemination of hadith, our method provides a reliable way of locating the activities of hadith transmitters in time, the vast majority for whom we have no historical record of when they lived. Given the importance of the temporal element in dating hadith for research on the history of early Islam and Islamic thought, our results are significant for this scholarly community as well. In addition, trophic levels makes the static aggregated hadith social network capable of diachronic analysis that allows scholars the ability to track change and continuity in the nature of the hadith preservation and transmission as represented in the Hadith Social Network.

Trophic levels provide a continuous measure of hierarchy for the nodes of a directed graph and illustrate that if a quantity was flowing through a graph it would flow from nodes of a lower trophic level before reaching nodes of a higher trophic level. This produces a well-defined continuous measure for vertices based on their distance from a source. In the case of the Hadith Social Network we have a flow of information from the source of statements, the Prophet Muhammad, and thus the hierarchical distance from the Prophet is congruent to a temporal distance because of the underlying transmission process and its associated dynamics. Trophic levels are indicative of a node's function in a directed flow process. In their original ecological context, the food web is abstractly represented by a network model where species represent nodes and edges represent predation relationships. Concurrently, the edges also indicate the direction of energy flow, from prey to apex predator, through the complex system. This is compactly defined by equation 1 where s represents the trophic level, d represents the in-degree and a the corresponding elements of the adjacency matrix for the respective node i (Johnson and Jones 2017). Basal nodes, nodes with an in-degree of zero, have a trophic level of 1 by definition.

$$\begin{aligned} s_i &= 1 + \frac{1}{d_i} \sum_j a_{ji} s_j, & \text{if } d_i \neq 0, \\ s_i &= 1 & \text{if } d_i = 0. \end{aligned} \quad (1)$$

In the ecological context, a basal vertex represents a primary producer species, for example grass. These are the species that provide energy into the food web and are thus the energy sources in complex ecosystems. The trophic level of every other species is 1 plus

the average trophic level of the species it eats. Trophic levels allow one to assign a height to each node and provide a vertical hierarchy and stratification by function. We shift our point of view from energy flow to information flow. Analogously the basal node, or primary producer, in our case is the Prophet, as each hadith has this node as its source, because every hadith, by definition, pertains to the Prophet.

Background and related work

Trophic analysis reveals the hierarchical structure of the Hadith Social Network and thereby makes its temporal dynamism explicit. A feature that is very prominent in this hadith network, hierarchy, often presents as asymmetry. This often has a crucial effect on a network's dynamical stability and contains much implicit information on the network's structure and properties. Processes such as percolation, stability and the range of dynamical states the system can explore are impacted (Asllani et al. 2018; Harush and Barzel 2017). The reason why trophic analysis is able to uncover temporal structure for the Hadith Social Network is because it relies on this feature of the network. Alongside sequential structure, these properties, especially prominent in this dataset, are exploited to reveal a partial order that is congruent to temporal structure and thus infer temporal information.

Our review of the literature on historical social networks yielded few studies that attempt to infer temporal information solely from network structure. Holme et al. (2015) provide a comprehensive review of modern temporal network theory but do not address a scenario comparable to the problem presented in this manuscript. Nevertheless there are some related studies. Early work in this area was done by Kempe et al where they define a typical inference problem in the temporal network space (Kempe et al. 2002): the problem of inferring if a time respected path exists between certain sets of nodes. Directed contact networks is another close analogue (Cybenko and Huntsman 2019) in the literature. They consist of a directed network with a temporal element and the constituent element is known as a contact. This is made up of a unique ordered triple: source, target and time. In our case, we do not have the time of interaction. The generalised trophic level provides a temporal structure and thus potentially an approximate transmission time span for the individuals that make up the network. Thus, this is a crucial first step in reconstructing an approximate directed contact network for this hadith dataset.

Recent work has also tackled a similar problem by reverse engineering the evolution of a dynamic network (Sreedharan et al. 2019). In this work, the authors develop a method to recover the original temporal order of nodes for a dynamic network in terms of a partial order by casting the problem as a rational linear integer program and solving this optimisation problem. Although this is conditional on a presupposed network growth mechanism, which is the preferential attachment model. Research along comparable lines is presented in Young et al. (2019), where again they presuppose the preferential attachment model and then utilise a maximum likelihood approach in reconstructing the network history. The main drawback of these methods is the computational cost in implementing them and also the discrete nature of the temporal order inferred in comparison to the continuous measure generalised trophic analysis produces. Parallel work has also been attempted on the more typical citation networks. In Clough et al. (2015)

the authors use a technique they introduce called transitive reduction. Transitive reduction prunes the citation network to only contain the causally relevant edges which they aptly name the causal skeleton of the citation network. In this case, the causal structure is uncovered by pruning away the edges which are not causally important in the citation chain. In the Hadith Social Network all the edges present are causally relevant, so transitive reduction would not be suitable because of the sequential structure of the network that comes about due to the nature of oral transmission. Additionally, we are inferring temporal information not just the causal structure.

Our review of the field of sociohistorical network analysis similarly yielded few academic studies. Only recently have historians come to utilise social network analysis as a method for the analysis of premodern societies (Rivers et al. 2011). One study describes the political parties and the associated elite network of families in medieval Florence (Padgett and Ansell 1993) that were related through marriage and economic relationships. The authors undertook a social network analysis to discover who the most influential families were. Similarly, archaeologists frequently deal with relationships between artefacts. Network science offers a way to study these relationships while also providing a visualisation of these relationships for exploration and communication purposes (Collar et al. 2015). One study, which defines nodes as cities and edges as trade, migration, or flows of cultural influence between them, tries to discover and measure migration or trade networks by tracking artefacts (Knappett 2013). The network archaeology space is potentially quite ripe for the application of trophic methodology in order to provide new insight in regards to historical networks (Lemercier 2015). Because archaeology constructs networks based on material artefacts, only in a limited number of cases can it track and measure the interaction of historical individual human beings (Mills 2017). Another potential area ripe for the application of trophic analysis involves religious networks. We found one study which uses network analysis to study the spread of religious ideas (Collar 2007). Finally, there exists only one work that attempts to conceptualise the history of hadith preservation and transmission as a social network (Senturk 1998). But this work does not create a graph drawn from hadith texts, nor take it beyond treating hadith transmission at the qualitative level as a network.

Material and methods

Description of hadiths and a short history of their development

In the early 8th century, shortly after death of the Prophet Muhammad, there emerged a decentralised network of scholars devoted to the collection, preservation, and dissemination of hadiths. Today and for centuries in the past, hadiths have been and are ubiquitous in Muslim culture: religious scholars use them to justify rules and core theological beliefs, preachers employ them in sermons, and parents use them to teach children. These narratives, found in a variety of different types of literary sources, number in the hundreds of thousands, and their significance to Islam, a religion with 1.8 billion adherents, is hard to overestimate. One feature of hadiths is the list of names, known as

an isnad, documenting the transmission of each narrative from one person to the next until it was written down by a systematic collector. Here are some examples of hadiths¹:

1. Bukhārī (d. 256/870)² writes: ‘Ubaydallāh b. Musā narrated to us, saying: Ḥanzala b. Abī Sufyān reported to us about ‘Ikrima b. Khālid: about Ibn ‘Umar, may God be pleased with them both, that he said that the Prophet, peace and blessings be upon him, said: “Islam is built on five things: testifying that there is no god but God and that Muḥammad is the messenger of God, establishing daily prayer, giving the charity tax, the pilgrimage, and fasting in Ramaḍān.”

2. Muslim (d. 261/875) writes: Sahl b. ‘Uthmān al-‘Askarī narrated to us: Yaḥyā b. Zakariyyā narrated to us: Sa’d b. Ṭāriq narrated to us, saying: Sa’d ‘Ubayda al-Sulamī narrated to me about Ibn ‘Umar: about the Prophet, peace and blessings be upon him, that he said: “Islam is built on five things: that God be worshiped and others beside him be rejected, establishing prayer, giving the charity tax, making the pilgrimage to the holy house, and fasting in Ramaḍān.”

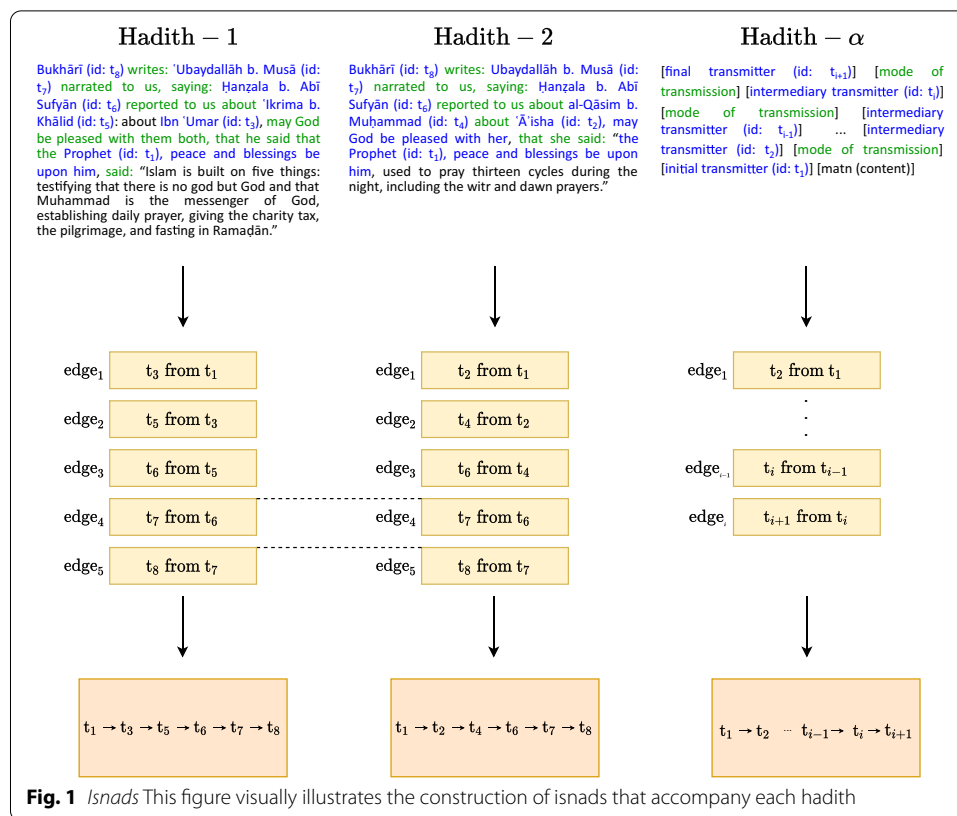
3. Ṭabarānī (d. 360/971) writes: Muḥammad b. Aḥmad b. Ḥammād, Abū Bishr al-Dūlābī, narrated to us in Egypt: my father narrated to us: Ash’ath b. ‘Aṭṭāf narrated to us about ‘Abdullāh b. Ḥabīb b. Abī Thābit: about al-Sha’bī: about Jarīr b. ‘Abdullāh al-Bajalī: about the Prophet, peace and blessings be upon him, that he said: “Islam is built on five things: testifying that there is no god but God, establishing prayer, giving the charity tax, [making] the pilgrimage to the holy house, and fasting in Ramaḍān.” Only Ash’ath and Sawra b. al-Ḥakam al-Qāḍī have narrated this hadith from ‘Abdullāh b. Ḥabīb.

As one can see, each of the three hadiths above is written down by three different systematic collectors from the ninth and tenth centuries and contain two basic types of data: the quotation of what the Prophet actually said, and the isnad. The isnad documents the transmission history from one individual narrator to the next until finally documented by the systematic collector. The three isnads of these three hadiths, contain thirteen unique narrators, some of whom occur in two of the isnads above. Figure 1 visually demonstrates how the isnad is constructed. The isnad is the key object of the dataset which allows us to construct the Hadith Social Network.

Some of the most well-known of these collections contain thousands of hadiths, and there exist hundreds of such collections, some of them regarded as more authoritative than others. Given the quantity of hadiths ultimately recorded in hadith collections, the number of distinct names mentioned in the isnads reaches the tens of thousands. In parallel with the endeavour to systematically collect hadiths circulating in Muslim society in the first three centuries of Islam in formal written form, there emerged a related endeavour to collect biographical information on the individual narrators whose names populated the isnads of hadiths. This activity was motivated by the needs of specialist hadith scholars that scrutinised the isnads in order to make judgements about the hadiths’ authenticity and the Muslim scholars’ more general desire to record and document the lives of notables in their societies (Brown 2009).

¹ The examples are taken from (Syed et al. 2019, 6–7)

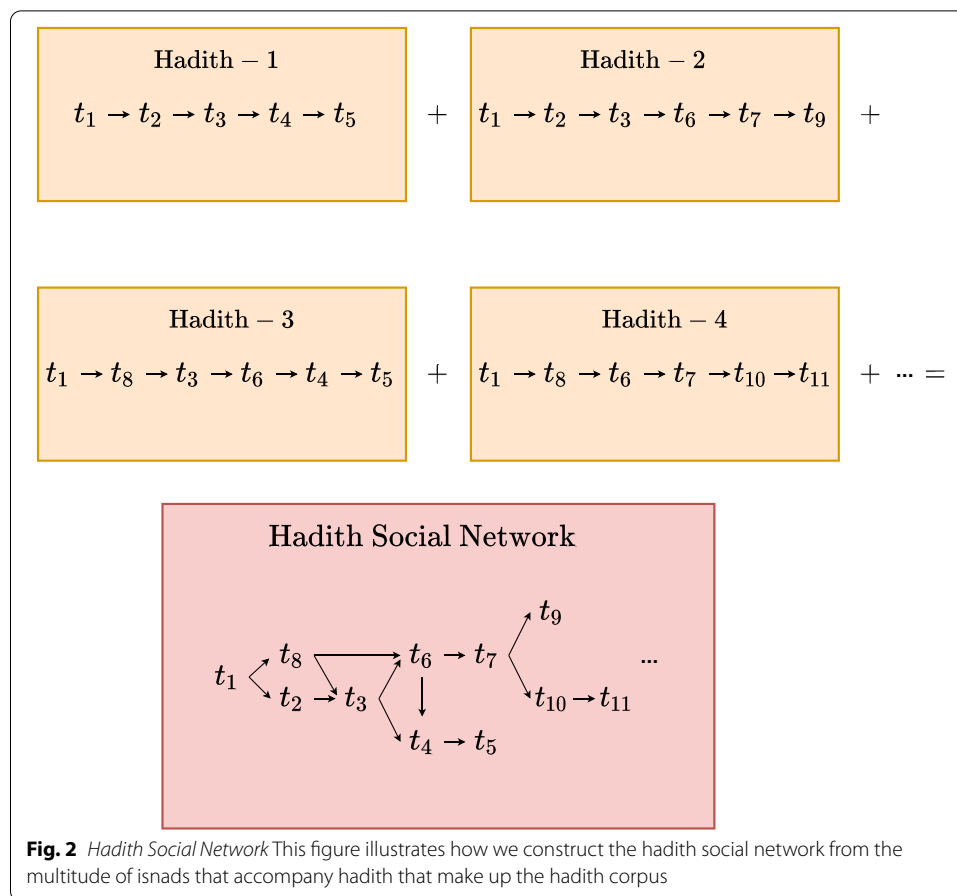
² The first number in the 256/870 is the year in the Islamic hijri calendar, the second is the year according to the Gregorian one.



Modern digitisation of hadiths and the construction of the hadith social network

In recent decades, both the collections of hadiths and the sources containing the biographies of narrators found in them have been digitised. One particular digitised compilation, Gawāmi' al-Kalim (GK), contains the most extensive such corpus of Sunni hadith collections. In addition to the sheer scale of GK's digitisation efforts, one other feature of its corpus separates it from other digitised corpora: it distinguishes the names of individual transmitters in the isnads, disambiguates them, and links them to a separate table containing biographical data gleaned from the biographical dictionaries. GK's collection contains 447,205 hadiths, citing 4,918,709 narrator names in the isnad, of which GK identified 49,819 as distinct individuals (Syed et al. 2019). GK's impressive digitisation efforts have laid the groundwork for approaching hadith as a rich and extensive temporal and spatial system with quantifiable properties amenable to statistical and computational analysis. One way to do this is to conceptualise the narrators found in all of the isnads as constituting a unified directed graph. The narrators are the nodes and the instances of hadith transmission from one narrator to the next are the directed edges. The edges can be weighted by the number of distinct hadiths transmitted between narrators. We used GK's data to construct such a graph. This graph, which we call the Hadith Social Network, consists of 49,309 nodes (narrators), 294,303 distinct directed edges (narrator relationships), and spans 9 centuries.

The directed network formed from the transmission activity of the narrators is the substructure for an information diffusion process and how this is constructed from isnads is illustrated in Fig. 2. Unlike many social networks, this one is diachronic and spatial with



thousands of individuals transmitting hadiths over the wide geographic expanse of lands under Muslim political control. One of the main tasks in determining hadiths' reliability is trying to determine the time period and geographical region in which transmission happened between individuals named in isnads. We now demonstrate the effectiveness of the trophic method in interpolating temporal information by leveraging the hierarchical and sequential structure of the graph.

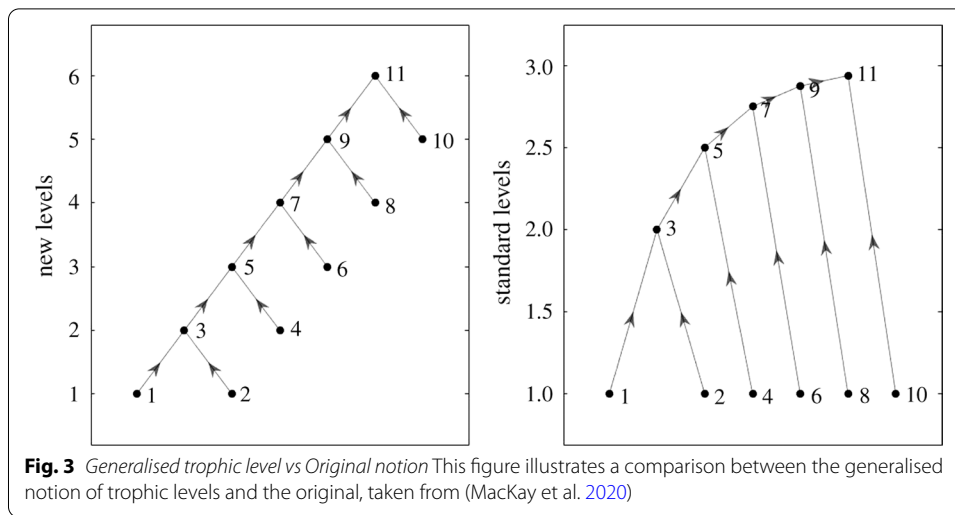
Generalised trophic analysis

We interpret this network as an information transmission network or an oral citation network. A citation network is a network where an edge represents the citation between two papers. In our case it represents the instance of a mostly oral transmission by a transmitter that was heard in person by the hadith's recipient. In both networks, a directed link is present between two nodes when a citation has been made. However, because the Hadith Social Network is largely an oral citation network, a consequence of the cultural value that Muslim societies came to place on the practice of hadith transmission, this leads to a distinct network structure because of the speed and dynamics of information transmission in the premodern age. To be authorised to cite one's source for a hadith, one typically met said narrator and heard the narrative from the narrator directly. Clearly this could only be done if the source narrator was still alive. Consequently, there

is an implicit order and sequential structure which the nodes and edges must take into account and manifests as a temporal constraint on the structure of the Hadith Social Network. This is in contrast to modern citation networks where citations can be made in a much more rapid manner and without the need for physical contact. One could cite an author even once they have passed away. Another key difference is that modern citation networks are well dated whereas in the hadith network this temporal information is not as specific as what is found in modern citation networks and is often incomplete or missing. Trophic analysis, by revealing a network's hierarchical flow structure, is well suited to studying the temporal aspect of the Hadith Social Network. We look to extract this temporal structure, by utilising the Hadith Social Network's intrinsic hierarchy, to shine a light on the nature of hadith transmission and infer further temporal information by leveraging the sequential topology of the network.

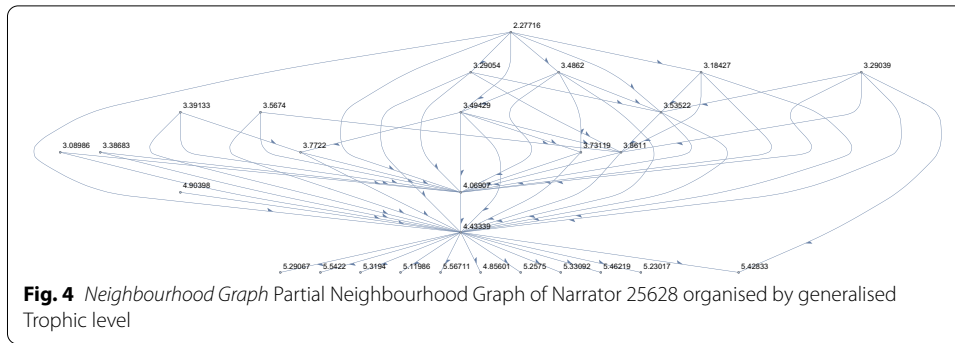
Furthermore, revealing the Hadith Social Network's temporal structure is instrumental for historians and scholars of hadith literature, because hadith criticism, authentication, and dating requires two fundamental pieces of data: when did each instance of hadith transmission happen and where. Here we run into a problem. When available, GK has done a good job of collecting the birth and death years of narrators and the cities in which they transmitted hadith. But, the premodern biographical sources that GK relied on only provide such information for a subset of hadith transmitters. Out of ~49,000 narrators, GK recorded death years for 14,149. For these 14,149 narrators, GK provided birth years for 4,697 transmitters. In addition to birth and death years, GK also categorised 39,595 narrators into one of 38 "generations". As we can see, the birth and death years provide relatively precise temporal information but cover a limited number of narrators, while the generations temporal data covers many more narrators though in a less precise manner, and it does not cover everyone in the graph. Moreover, GK provides no information on their method for placing narrators in a given generation. Death and birth years and generation numbers are pieces of temporal information that are extrinsic to the Hadith Social Network itself. Yet, because isnads are sequences of narrators that transmitted information over time, they contain implicit temporal information (Alkaoud and Syed 2021). Adding all of the isnads into one cohesive graph, aggregates all of a given narrator's sources and recipients, thereby potentially providing a temporal location for all of Hadith Social Network's individuals.

The network we have constructed is an aggregated snapshot of an event-based network (Masuda and Lambiotte 2016), where an event constitutes the meeting of two scholars to transmit information. The event sequences are bursty when considering the time span of the dataset and the dynamics of hadith collection and narration. Consequently, this network exhibits strong parallels with a directed social contact network (Cybenko and Huntsman 2019; Holme 2005) but with only partial temporal information available. Trophic analysis will render a temporal order, that is strongly congruent to a hierarchical order. We then leverage the sequential structure to allow us to interpolate temporal positions, as we illustrate later. This permits scholars to determine further temporal information around relationships and interactions and thus trace information flow from the Prophet to subsequent narrators. This allows one to potentially reconstruct the event based temporal network from the static aggregated snapshot as well as temporal network metrics like temporal paths, walks and centrality measures.



The original formulation of trophic levels applies only to networks with basal nodes. Until recently trophic analysis was restricted to this subset of graphs because of the restrictions its mathematical definition imposed. This has been recently generalised, with multiple approaches, see MacKay et al. (2020), Moutsinas et al. (2021), to account for networks which do not contain basal nodes. The two new methods have been used to study the spread of disease, economic and trade networks, linguistic networks, and gene and biological networks among others. The GraphHierarchy (Moutsinas et al. 2021) method emphasises influence and information flow of vertices on the dynamics of the networks in its hierarchical analysis of the network. It applies equally to undirected and directed networks. In the directed case there are two alternate perspectives, control and dependence. These are captured by looking at the graph and its transpose, thus capturing asymmetry between the two perspectives. This is compactly represented by the forward and backward hierarchical levels. Out of this new method, metrics like democracy coefficient and influence centrality have been devised as well as a symmetric notion of hierarchical levels. The approach laid out in MacKay et al. (2020) is a little different and only applies to directed networks. We utilise this novel MacKay et al. generalisation to tackle the problem we present with the hadith network (MacKay et al. 2020). This generalisation was utilised over the method outlined in Moutsinas et al. (2021) due to its ranking of basal nodes in a sequential and integrated manner as illustrated in Fig. 3 and this is also made it more suitable in comparison to the restricted notion of trophic levels, which ranks basal nodes equally.

The dynamic flow of information transpiring on this network makes the trophic method ideal. We have information flowing from a source, The Prophet Muhammad, cascading sequentially from one transmitter to the next, through the generations of hadith narrators. Narrators from the same generation will cluster within the same time period and so anyone close to them in the trophic position will be a contemporary and thus have a similar generalised trophic level. The very fact that transmitters are more likely to interact with the next generation through the formation of teacher-student relationships means the inherent structure of the network is hierarchical and sequential; rendering trophic analysis particularly suitable to infer temporal information. In effect



we have a partitioning of the nodes into the different sequential stages of information transmission, which in this case are temporal stages from one generation of transmitters to the next. This partitioning also aids visualisation of the hadith network as we can hierarchically, and thus temporally, organise its structure, helping to improve potential analysis. This is shown in Fig. 4. For each node $n \in N$, one can define its in-strength, which represents the number of unique source relationships a hadith scholar had, and out-strength, which represents the number of unique transmission relationships a hadith scholar had. Then by using the components of the weighted adjacency matrix, w_{mn} , one can define these two terms mathematically:

$$w_n^{in} = \sum_{m \in N} w_{mn} \text{ and } w_n^{out} = \sum_{m \in N} w_{nm} \quad (2)$$

Then one can define the total weight of the node n , which represents the sum of an individual's unique transmission and source relationships, by the vector u_n :

$$u_n = w_n^{in} + w_n^{out} \quad (3)$$

and the imbalance of the node n , which represents the difference between the number of unique transmitter and source relationships a hadith scholar had, by the vector v_n

$$v_n = w_n^{in} - w_n^{out} \quad (4)$$

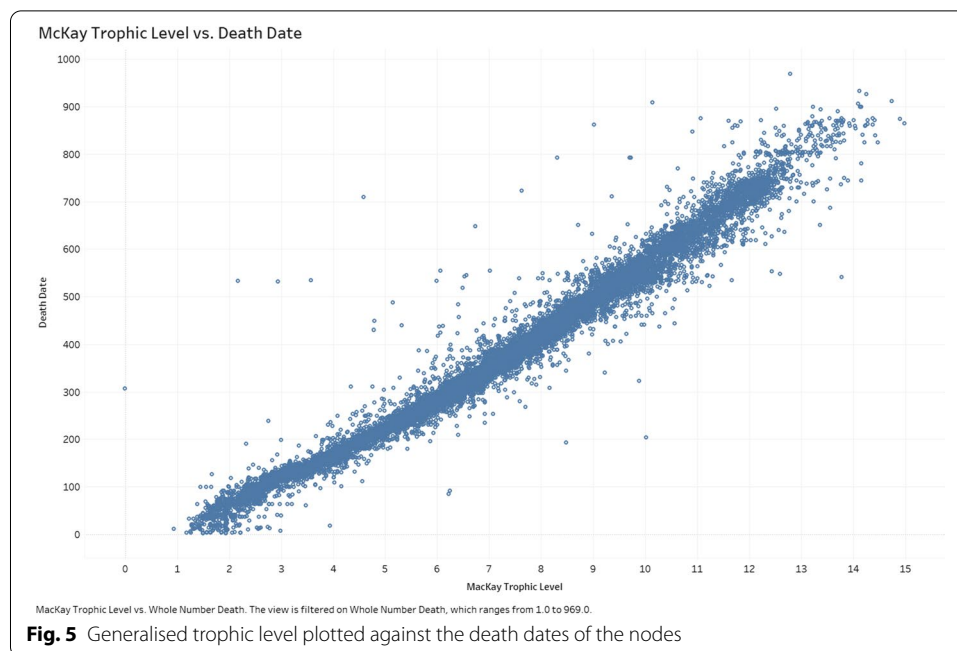
The imbalance of the node, v_n , represents the difference in information flow for a hadith narrator/source. The weighted graph-Laplacian operator is defined by:

$$\Lambda = \text{diag}(u) - W - W^T \quad (5)$$

The improved notion of trophic levels can be written as the below solution, h , of the linear system of equations. For further details on this improved notion of trophic level we refer the reader to MacKay et al. (2020). Equation 6 is easily and efficiently solved through modern linear algebra packages and so there is no problem in utilising this on the vast Hadith Social Network.

$$\Lambda h = v \quad (6)$$

In essence, the improved notion of trophic levels partition the network into sequential temporal generations of hadith narrators. The Hadith Social Network exhibits a

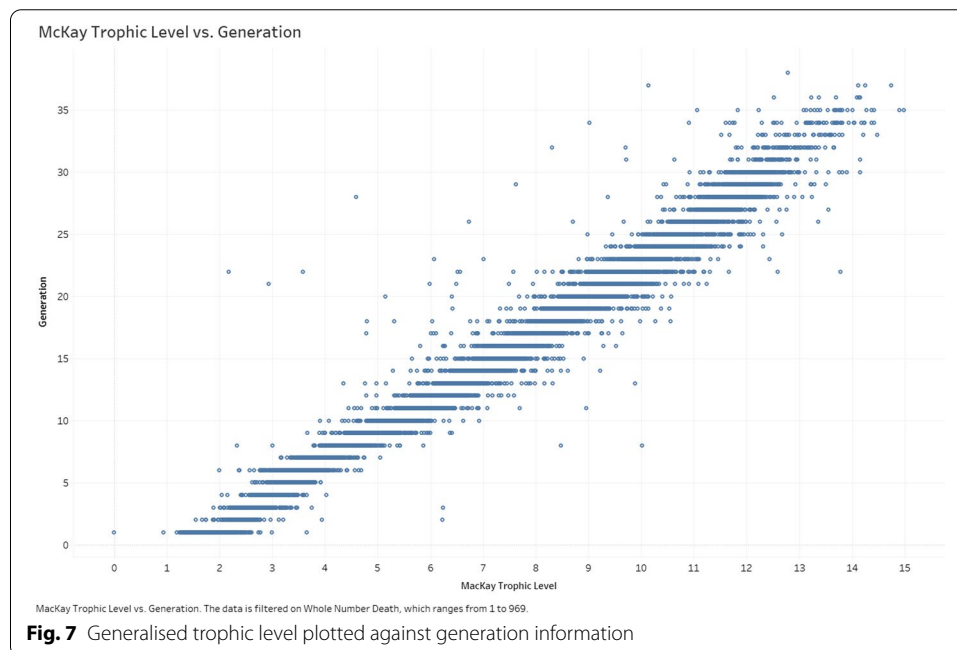
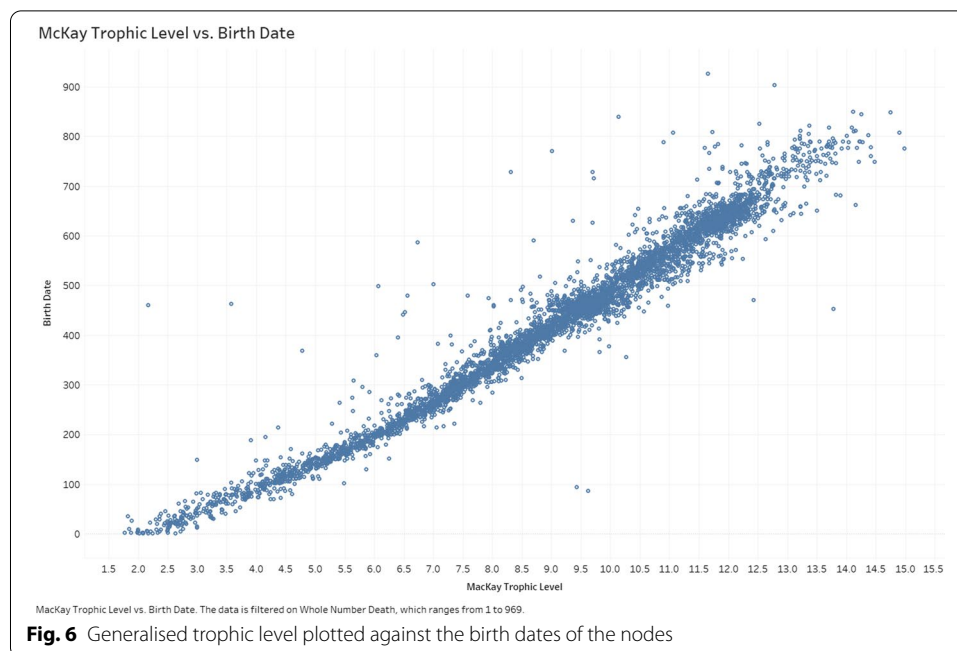


sequential structure that makes its hierarchy more salient and coherent than what is found in a typical citation network. Thus these properties would allow one to infer the evolution of this dynamic network using the trophic methodology. As we will now go on to show in our results, the improved notion of trophic levels correlates excellently with the partial temporal information, extrinsic to the network, that we do have and therefore we can use trophic levels to infer a continuous temporal measure and thus provide a temporal position for the vast majority of the individuals for which we have no temporal information.

Another reason the trophic framework is fitting for the problem at hand is because as hadith transmission activity proliferated in the different cities and became a recognisable practice, narrators sought to minimise their temporal distance from the Prophet by seeking the oldest scholar they could find. Collectors would actively seek out scholars much older than them as this would shorten their distance to the Prophet (Davidson 2020, 66–75). Collectively the system evolved to optimise the individual agent's distance from the information source, the Prophet, as this gave the narrator great renown amongst their peers. This feature influences the dynamics of transmission and is accounted for in the trophic framework. The distribution of age differences between narrator and student is narrower than what would be expected at random. Thus there is more likely to be a narrow range of values for a target height difference which makes the trophic method especially suitable.

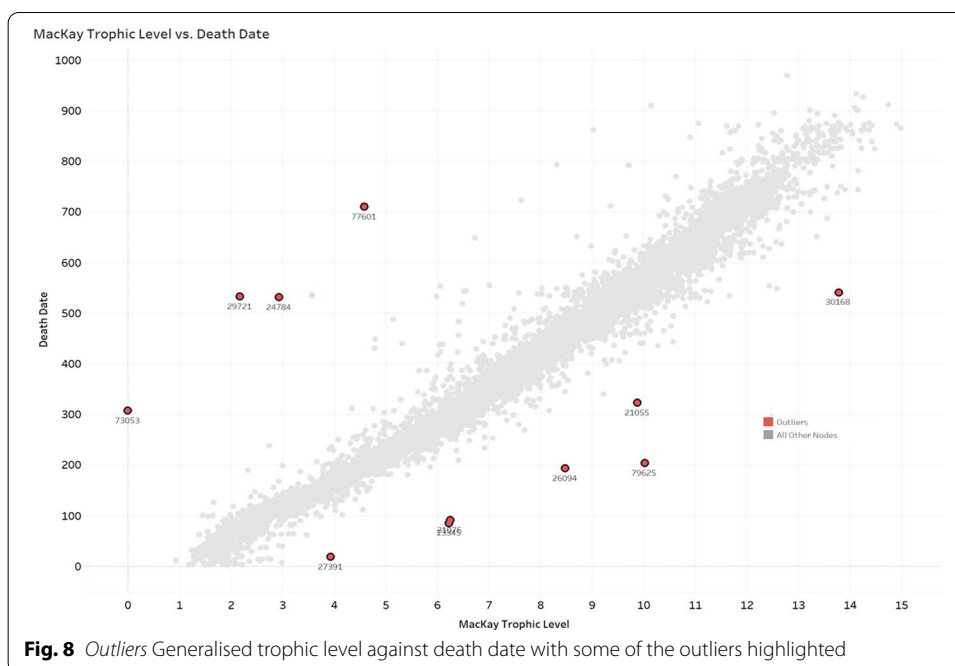
Results

Generalised trophic level, by supplying a continuous temporal measure, provides us with much more temporal information compared to the minimal and opaque generation information and scant death and birth dates that the GK data provides, allowing scholars to probe the evolution of the complex system in a much more precise manner. We



test the effectiveness of this method by comparing the generalised trophic levels against the death dates, birth dates, and generation numbers for those nodes for whom we have such data. As can be seen in Fig. 5, for the vast majority of nodes there is a strong direct correlation between the narrators' death years and their generalised trophic level.

The same strong correlation holds between narrators' birth years and their generalised trophic levels as illustrated in Fig. 6. The ground truth is a labelling done by the dataset



providers who say they utilised biographical information to give a generation number to each individual. Now if we compare the bins produced by the trophic level against the number of generations, we can see a striking similarity illustrated through Fig. 7. Given the correlation of the generalised trophic levels with birth and death years and GK's generation assignments, we are confident that they convey temporal information for a much greater number of narrators than birth/death years and at a more precise and justifiable basis than GK's generation assignments. In this clearly complex network of ~49,000 nodes and ~300,000 edges, we have derived a continuous ranking score in a hierarchy where the score reflects in essence how far one is from the source, the Prophet. This we have shown to be strongly correlated with a temporal distance that conveys the temporal interval that has elapsed since the time of the Prophet. Because it is not just an ordinal ranking but a continuous score, it is more congruent to a temporal proxy. The generalised trophic level is essentially providing a non-linear function of time. We have the final snapshot of what is clearly a dynamical network and we have, in essence, constructed a temporal dynamic picture of a static network structure utilising the aggregated data. This potentially allows for a huge reduction in human effort in dating and authenticating hadith, which partly relies on temporal data to make judgements about a purported isnad's veracity.

Historians and hadith scholars have long known that there were mistakes made in the transmission of hadiths and these can appear in the isnads and not just the content of the hadith. The digitisation of the hadith corpus may also introduce mistakes in a variety of ways, many of which are documented in Syed et. al's work (Syed et al. 2019). Generalised trophic levels allow us to highlight potential errors. While there is a high degree of correlation between these extrinsic measures and generalised trophic levels, there are a handful of nodes for which there is no correlation as seen in Fig. 8. We manually investigated the underlying isnad and biographical data to see if the cause of the outliers is

errors in GK's data or our processing of it. We identified 12 nodes as extreme outliers. We were able to confirm that two of the nodes belonged to erroneous edges. In one case, GK had misidentified a narrator in the isnad and assigned him the wrong narrator ID (ID 73053). In another case, we found that our processing of GK's text file introduced an error (ID 77601). In the nine other cases, we found reasons to strongly suspect errors in GK's dataset. In two cases the differences in death dates involving the outlier with another transmitter in an edge was abnormally large, indicating a case of a misidentified narrator in a hadith text which would naturally generate an inaccurate generalised trophic level for this node and cascade through to the other nodes. In four other cases the differences in the generations either involving the outlier with another transmitter directly, or in an edge one hop removed from the outlier, was abnormally large. Only in one case (ID 27391) did an initial investigation not reveal a *prima facie* reason to suspect error. The investigation of the outliers revealed the utility of generalised trophic levels as a method to probe and clean the data. The generalised trophic levels summarised the narrator's placement in temporal space, because it positions the nodes according to their integration in the overall flow hierarchy of the network, which we then used to compare with externally labelled data.

We constructed a temporal structure from the static network description using generalised trophic levels, which is a hierarchical measure of nodes that strongly correlates with a temporal distance from the Prophet in the Hadith Social Network. The temporal structure allows us to determine temporal position of all of the network's narrators, giving us a tighter bound on the temporal information of the transmission of statements between two individuals. This paves the way for a partial reconstruction of the event based temporal network from the static aggregated snapshot. Inferring dynamic events from a static network description would be the converse of what is typically done with temporal networks, which involves trying to find a sort of static description to be able to keep track, and make sense, of the temporal network (Holme and Saramäki 2019). This further highlights the novelty of the work. This historical process is a complex interplay of singular personalities interacting according to their own aims and beliefs which manifests as an emergent phenomenon producing the body of knowledge that is the hadith corpus. We have now opened up the study of the process of the flow of information between hadith transmitters and shed light on the historical process of hadith transmission and preservation. We can now use this temporal information to carry out more detailed analyses by using the generalised trophic level as a temporal proxy and subsequently focus on certain historical periods, which is an indispensable capability for the Islamic studies community.

Impact and outlook

Generalised trophic analysis provides a novel tool in the study of complex systems rendering the inherent directed flow structures found in economic and social systems interpretable and useful, thus revealing new insights. The generalised trophic level also aids visualisation of networks with an inherent hierarchical structure in an efficient manner as illustrated in Fig. 4. Visualisation of large directed networks is a challenging task which is computationally difficult, for which only sub-optimal algorithms (Carmel et al.

2004) exist. Trophic levels provides another perspective by which to visualise networks, particularly networks which have an underlying hierarchical structure.

Trophic levels serve as useful temporal metrics in this case study and having this temporal information will open up new avenues of investigation for scholars of Islamic studies. Historians may systematically and quantitatively consider things such as how the volume of hadith transmission changed over time and place or whether the transmission of particular themes flourished in specific cities or periods or quantitatively determine the prominence of different narrators (e.g. centrality metrics) in different eras. Traditional scholars of hadith and historians can use these considerations to develop new methods of hadith authentication and dating and to discover the criteria that great hadith scholars of the past used to discriminate between credible, defective, and fabricated isnads. Scholars can now explore and analyse the network in a dynamic sense by using the generalised trophic level as a temporal proxy. Scholars could then combine this with a social network analysis to help identify influential narrators and detect communities for different time periods of the network's history and compare this with information on the larger social forces found in other historical sources. Scientists can also incorporate trophic level temporal information into a Bayesian inference model to ascertain a high confidence direct relationship that is statistically sound and incorporates uncertainty bounds within the model, thus providing more direct predictions on the birth and death years of the hadith narrators who make up the hadith network. This would be invaluable for hadith scholars who are looking to probe the historicity of hadith transmission and early Islam further. This would also lead to the eventual partial reconstruction of an event based temporal network from the static aggregated snapshot.

Another concept within the trophic framework that could be utilised for further analysis is trophic incoherence. Within the ecological context, in a perfectly layered food web, all species have integer trophic levels and the difference between the trophic levels of the prey and the predator is 1. In practice, this rarely happens and the notion of the trophic incoherence parameter was introduced as a way to measure how far a food web is from being perfectly layered. It is a measure of how close a network's structure is to an ideal food web; a partition of the network into distinct integer trophic levels. The closer to the ideal structure the more stable the dynamic of the spreading process, which then is also a measure of dynamical stability and how directed a network is Johnson et al. (2014). The trophic incoherence could provide us with a new perspective on the global structure of networks, showcasing and quantifying their hierarchical nature. Trophic incoherence may tell us whether or not the Hadith Social Network exhibits a graph structure that has a stable dynamic and was conducive to a more reliable/authoritative transfer of information. Previous research has shown that non-normality of networks improves information transmission (Baggio et al. 2020; Baggio and Zampieri 2021) and non-normality is highly correlated with incoherence (Johnson 2020). Therefore it could provide more insight on the fidelity of the information transmitted through the Hadith Social Network in a quantitative manner, which is a pertinent and highly important question in the field. It also quantifies how present cycles and feedforward motifs are in the network structure. In the case of the Hadith Social Network it would reveal potential cycles between narrators close to each other in the hierarchical/temporal measure and are therefore highly likely to be contemporaries.

As a network that lasted almost a millennium, the practices that constituted the Hadith Social Network and their social meanings, along with the institutions that supported them changed over time. Two recent studies that rely on qualitative narrative data document the nature of this change. Schoeler documents the eventual formalisation of hadith transmission activities that combined oral and written transmission in the first three centuries of Islamic history (Schoeler et al. 2006). For Schoeler and other hadith critics, one particularly influential person is a scholar named Ibn Shihāb al-Zuhri (d. 124/741). He is considered the first systematic collector and disseminator of hadith. Whereas scholars have largely relied on anecdotes preserved in literary sources to come to this judgement, we can use quantitative measures such as trophic coherence to detect if there is a difference in hadith transmission activity in the system after him. Since al-Zuhri died in 124/741, and he had largely inaugurated and crystallised a new method of hadith collection and dissemination by then, we should see the nature of the transmission change dramatically at this time, which should lead to a dramatic phase change in the network's structure and topology. Using trophic analysis we could provide a quantitative confirmation or invalidation of this hypothesis on hadith transmission formalisation. Whereas Schoeler focused on hadith transmission in early Islam, Garrett Davidson examines the function and social meanings of hadith transmission after the fourth/tenth century and finds a sharp shift after the publication of what came to be regarded as authoritative written hadith collections (Davidson 2020). Trophic incoherence could detect where there is a noticeable change in the network structure which correlates with known historical information. In addition to aiding literary anecdotal sources in describing changes in hadith transmission practice, trophic analysis of the Hadith Social Network's structure may aid in judgements about whether it is more or less conducive to the spread of rumours. We could analyse if there is a signature network structure which aids rumour propagation like in the early years of Islam when civil war was rife and later on when the study of hadith became formalised and thus transmissions were made and recorded in a more rigorous manner. This could then potentially be used in other rumour and information spreading networks as a way of determining if the network structure was conducive to an accurate transfer of information or prone to instability and hence facilitate the emergence of misinformation.

Overall we propose that the trophic approach can be used as a methodological strategy to explore new patterns and structures for a variety of economic and social science datasets, particularly those with an underlying directed, hierarchical or asymmetrical structure (Pilgrim et al. 2020). Citation networks such as scientific (Zeng et al. 2017), legal (Fowler et al. 2007; Hoadley et al. 2021) and patent (van Raan 2017) networks would be prominent examples. Whereas, in the case of the Hadith Social Network the hierarchical structure manifests as temporal structure. This is due to the particular features of information transmission in the premodern age, in legal, scientific or patent citation networks the trophic levels could manifest as influence measures as they do in the ecological scenario and thus could be used to measure impact of publications, legal rulings and innovative technologies. Complexity frequently presents as hierarchy (Simon 1991); examples span from neuroscience (Kaiser et al. 2010), biology (Ma et al. 2004; Yu and Gerstein 2006; Lagomarsino et al. 2007), urban networks (Batty and Longley 1994), the internet (Vázquez et al. 2002) and social and economic systems (Valverde and Solé 2007;

Mihm et al. 2010; Krackhardt 2014; Antràs et al. 2012). In the case of hadith transmission we have a hierarchical order that reflects a conspicuous temporal structure. In economic scenarios, the hierarchy can be induced by economic differentials and in social datasets hierarchy may be induced by the influence differentials between respective individuals or institutions. Trophic analysis offers a new perspective in probing these fundamental hierarchies that are present in the complex systems inherent to human life.

Acknowledgements

CS would like to thank Robert MacKay and Marya Bazzi for their invaluable advice and guidance. MS, DH, and NS would like to thank Behnam Sadeghi for his input into early conversations and analysis about the possibility that the Hadith Social Network's network structure alone could provide important temporal information.

Author contributions

CS came up with the idea of applying trophic analysis to extract temporal information after the problem was formulated by MS, NS and DH. CS and MS wrote up the manuscript with editing and visualisation input from DH. MS, NS and DH curated the data and contextualised it into the network format. The analysis was carried out by CS with input from MS. All authors read and approved the final manuscript.

Funding

CS gratefully acknowledges funding from the UK Engineering and Physical Sciences Research Council under the EPSRC Centre for Doctoral Training in Urban Science (EPSRC Grant No. EP/L016400/1). MS would like to thank University of California, Davis office of research for their new research initiatives grant, which partially funded this research.

Availability of data and materials

The dataset analysed during the current study is available in the Gawāmi' al-Kalim repository, https://archive.org/download/G_Kalim/G_Kalim.zip.

Declarations

Ethics approval and consent to participate

Not applicable.

Consent for publication

Not Applicable.

Competing interests

The authors declare that they have no competing interests.

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Received: 12 January 2022 Accepted: 30 April 2022

Published online: 18 May 2022

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